PART2　∑の性質

例題

次の和を求めよ。

(1) $\displaystyle\sum_{k=1}^{n}(2k+3)$　　(2) $\displaystyle\sum_{k=1}^{n}\left(3k^2+k-4\right)$　　(3) $\displaystyle\sum_{k=1}^{n}4k^3$

Point Pickup

練習

次の和を求めよ。

(1) $\displaystyle\sum_{k=1}^{n}(4k-1)$　　(2) $\displaystyle\sum_{k=1}^{n}(k-1)(k-5)$　　(3) $\displaystyle\sum_{k=1}^{n}(4k^3+1)$

第2講　PART2 確認問題

次の和を求めよ。

1 $\sum_{k=1}^{n}(4k-5)=n(\boxed{ア}n-\boxed{イ})$

2 $\sum_{k=1}^{n}(3k-1)(k+2)=n(n^2+\boxed{ウ}n+\boxed{エ})$

3 $\sum_{k=1}^{n}k(k^2+1)=\frac{1}{\boxed{オ}}n(n+\boxed{カ})(n^2+n+\boxed{キ})$

PART2　Σの性質

例題

次の和を求めよ。

(1) $\displaystyle\sum_{k=1}^{n}(2k+3)$　(2) $\displaystyle\sum_{k=1}^{n}\left(3k^2+k-4\right)$　(3) $\displaystyle\sum_{k=1}^{n}4k^3$

(1) $\displaystyle\sum_{k=1}^{n}(2k+3)=2\sum_{k=1}^{n}k+\sum_{k=1}^{n}3=2\cdot\frac{1}{2}n(n+1)+3n=n^2+4n=n(n+4)$

(別解)

$\displaystyle\sum_{k=1}^{n}(2k+3)=5+7+9+\cdots\cdots+(2n+3)=\frac{1}{2}n\{5+(2n+3)\}=\frac{1}{2}n(2n+8)=n(n+4)$

(2) $\displaystyle\sum_{k=1}^{n}\left(3k^2+k-4\right)=3\sum_{k=1}^{n}k^2+\sum_{k=1}^{n}k-\sum_{k=1}^{n}4$

$\displaystyle=3\cdot\frac{1}{6}n(n+1)(2n+1)+\frac{1}{2}n(n+1)-4n$

$\displaystyle=\frac{1}{2}n\{(n+1)(2n+1)+(n+1)-8\}$

$\displaystyle=\frac{1}{2}n\left(2n^2+4n-6\right)=n\left(n^2+2n-3\right)=n(n+3)(n-1)$

(3) $\displaystyle\sum_{k=1}^{n}4k^3=4\sum_{k=1}^{n}k^3=4\cdot\left\{\frac{1}{2}n(n+1)\right\}^2=n^2(n+1)^2$

練習

次の和を求めよ。

(1) $\displaystyle\sum_{k=1}^{n}(4k-1)$　(2) $\displaystyle\sum_{k=1}^{n}(k-1)(k-5)$　(3) $\displaystyle\sum_{k=1}^{n}\left(4k^3+1\right)$

(1) $\displaystyle\sum_{k=1}^{n}(4k-1)=4\sum_{k=1}^{n}k-\sum_{k=1}^{n}1=4\cdot\frac{1}{2}n(n+1)-n=2n^2+n=n(2n+1)$

（別解）

$$\sum_{k=1}^{n}(4k-1)=3+7+11+\cdots\cdots+(4n-1)=\frac{1}{2}n\{3+(4n-1)\}=\frac{1}{2}n(4n+2)=n(2n+1)$$

(2) $\displaystyle\sum_{k=1}^{n}(k-1)(k-5)=\sum_{k=1}^{n}\left(k^2-6k+5\right)=\sum_{k=1}^{n}k^2-6\sum_{k=1}^{n}k+\sum_{k=1}^{n}5$

$$=\frac{1}{6}n(n+1)(2n+1)-6\cdot\frac{1}{2}n(n+1)+5n$$

$$=\frac{1}{6}n\{(n+1)(2n+1)-18(n+1)+30\}$$

$$=\frac{1}{6}n\left(2n^2-15n+13\right)=\frac{1}{6}n(2n-13)(n-1)$$

(3) $\displaystyle\sum_{k=1}^{n}\left(4k^3+1\right)=4\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}1$

$$=4\cdot\left\{\frac{1}{2}n(n+1)\right\}^2+n$$

$$=n^2(n+1)^2+n=n\left\{n(n+1)^2+1\right\}=n\left(n^3+2n^2+n+1\right)$$

第 2 講　PART2 確認問題 解答

1 **[正解]** ア：2，イ：3

[解説]

$$\sum_{k=1}^{n}(4k-5)=4\sum_{k=1}^{n}k-\sum_{k=1}^{n}5 \quad \text{[A]}$$

$$=4\cdot\frac{1}{2}n(n+1)-5n \quad \text{[B]}$$

$$=2n^2-3n$$

$$=\boldsymbol{n(2n-3)}$$

[A]　$\sum$ を分けて表す。

[B]　$\sum_{k=1}^{n}c=cn$（c は定数）

$\sum_{k=1}^{n}k=\frac{1}{2}n(n+1)$

（別解）

$$\sum_{k=1}^{n}(4k-5)=(-1)+3+7+\cdots\cdots+(4n-5) \quad \text{[C]}$$

$$=\frac{1}{2}n\{(-1)+(4n-5)\}$$

$$=\frac{1}{2}n(4n-6)$$

$$=\boldsymbol{n(2n-3)}$$

[C]　初項 -1，末項 $4n-5$，項数 n の等差数列の和である。

2 [正解] ウ：4,　エ：1

[解説]

$$\sum_{k=1}^{n}(3k-1)(k+2)=\sum_{k=1}^{n}\left(3k^2+5k-2\right)$$

$$=3\sum_{k=1}^{n}k^2+5\sum_{k=1}^{n}k-\sum_{k=1}^{n}2$$

$$=3\cdot\frac{1}{6}n(n+1)(2n+1)+5\cdot\frac{1}{2}n(n+1)-2n$$ [D]

$$=\frac{1}{2}n(n+1)(2n+1)+\frac{5}{2}n(n+1)-2n$$

$$=\frac{1}{2}n\{(n+1)(2n+1)+5(n+1)-4\}$$

$$=\frac{1}{2}n\left(2n^2+8n+2\right)$$

$$=\boldsymbol{n\left(n^2+4n+1\right)}$$

[D] $\sum_{k=1}^{n}c=cn$（c は定数）

$\sum_{k=1}^{n}k=\frac{1}{2}n(n+1)$

$\sum_{k=1}^{n}k^2=\frac{1}{6}n(n+1)(2n+1)$

3 [正解] オ：4,　カ：1,　キ：2

[解説]

$$\sum_{k=1}^{n}k\left(k^2+1\right)$$

$$=\sum_{k=1}^{n}k^3+\sum_{k=1}^{n}k$$

$$=\left\{\frac{1}{2}n(n+1)\right\}^2+\frac{1}{2}n(n+1)$$ [E]

$$=\frac{1}{4}n^2(n+1)^2+\frac{1}{2}n(n+1)$$

$$=\frac{1}{4}n(n+1)\{n(n+1)+2\}$$

$$=\boldsymbol{\frac{1}{4}n(n+1)\left(n^2+n+2\right)}$$

[E] $\sum_{k=1}^{n}k=\frac{1}{2}n(n+1)$

$\sum_{k=1}^{n}k^3=\left\{\frac{1}{2}n(n+1)\right\}^2$